GABARITO Prova B3 — Geometria Analítica

TURMA A:

1. $l: (x,y,z) = (2,-1,3) + t(1,2,-1)$ e o plano $\pi: 2x-y+z=4$

(a) Na reta: $\left.\begin{array}{l} x = 2+t \\ y = -1+2t \\ z = 3-t \end{array}\right\}$ Sustituindo em π: $2(2+t)-(-1+2t)+(3-t)=4$

$4+\cancel{2t}+1-\cancel{2t}+3-t=4$

$8-t=4 \Rightarrow t=4.$

Logo, $\left.\begin{array}{l} x = 2+4 = 6 \\ y = -1+2(4) = 7 \\ z = 3-4 = -1 \end{array}\right\}$ $l \cap \pi = (6,7,-1) = I(x,y,z)$

(b) Posição relativa entre l e π:

$\vec{v}$: vetor diretor de $l \rightarrow \vec{v} = (\underset{m}{1}, \underset{n}{2}, \underset{p}{-1})$

Logo, $am+bn+cp \overset{?}{=} 0$

pelo plano temos $\left\{\begin{array}{l} a=2 \\ b=-1 \\ c=1 \\ d=-4 \end{array}\right\}$ $\pi: 2x-y+z-4=0$

$2(1)+(-1)(2)+1(-1)$

$2-2-1 = \underline{-1 \neq 0}$

$\Downarrow$

l e π são $\perp$

com $l \cap \pi = (6,7,-1)$

2. $P = (\overset{1,3,4}{\cancel{1,2,1}})$; $\pi: (1,0,0) + \lambda(1,0,0) + \mu(-1,0,3)$

(a) $\pi: \left\{\begin{array}{l} x = 1+\lambda-\mu \\ y = 0 \\ z = 3\mu \end{array}\right.$ $\rightarrow$ converter em Eq. geral do plano $\rightarrow$ $\vec{v}_1 = (1,0,0)$, $\vec{v}_2 = (-1,0,3)$

$$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 0\vec{i}, -\vec{j}(3-0), +\vec{k}(0) = (0,-3,0)$$

~~$\pi: ax+by+cz+d=0 \rightarrow \pi: 0x-3y+0z+d=0$~~

~~$-3y+d=0$~~

Usando $A(1,0,0) \rightarrow$ ~~$d=0 \therefore \pi: -3y=0$~~ ^ a Def. $\underbrace{d(P,\pi) = \left|\vec{AP} \cdot \frac{\vec{n}}{|\vec{n}|}\right|}$

$A \in \pi$.

$\vec{AP} = P - A = (1,3,4) - (1,0,0) = (0,3,4)$

$|\vec{n}| = \sqrt{0^2+(-3)^2+0^2} = 3.$

$d(P,\pi) = \left|\dfrac{(0,3,4)\cdot(0,-3,0)}{3}\right| = \left|\dfrac{-9}{3}\right| = \dfrac{9}{3} = \underline{\underline{3}}$

(b) $P=(1,3,4)$ e $\vec{v}=(2,3,2)$

$r: \dfrac{x-1}{2} = \dfrac{y-3}{3} = \dfrac{z-4}{2}$

$r: (x,y,z) = (1,3,4) + t(2,3,2)$

$\pi: ax+by+cz+d=0 \rightarrow \pi: 0x+3y+0z+d=0$

$-3y+d=0. \quad \therefore d=0$ (usando $A(1,0,0)$)

$\therefore \pi: -3y=0 \rightarrow \boxed{\pi: y=0}$ Eq. do plano

(c)

$r: \begin{cases} x = 1+2t \\ y = 3+3t \\ z = 4+2t \end{cases}$

Substituindo na Eq geral do plano: $\pi: y=0$, temos

$0 = 3+3t \rightarrow t = \dfrac{-3}{3} = -1 \rightarrow t=-1$

$\therefore \left.\begin{array}{l} x = 1+2(-1) = 1-2 = -1 \\ y = 0 \\ z = 4+2(-1) = 4-2 = 2 \end{array}\right\} r \cap \pi = (-1,0,2)$

3. (a) $F_2(0,6) \rightarrow F_2(0,c)$

$F_1(0,-6) \rightarrow F_1(0,-c)$

Centro $C(0,0)$

Eixo maior $\rightarrow 2a=34 \quad \therefore a=17$

$\downarrow$

Está sobre o eixo dos y.

Forma geral: $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1 \Rightarrow$

$a^2 = b^2+c^2$

$17^2 = b^2+6^2 \Rightarrow b^2 = 253 \Rightarrow b = \sqrt{253}$

$\boxed{\dfrac{x^2}{253} + \dfrac{y^2}{289} = 1}$

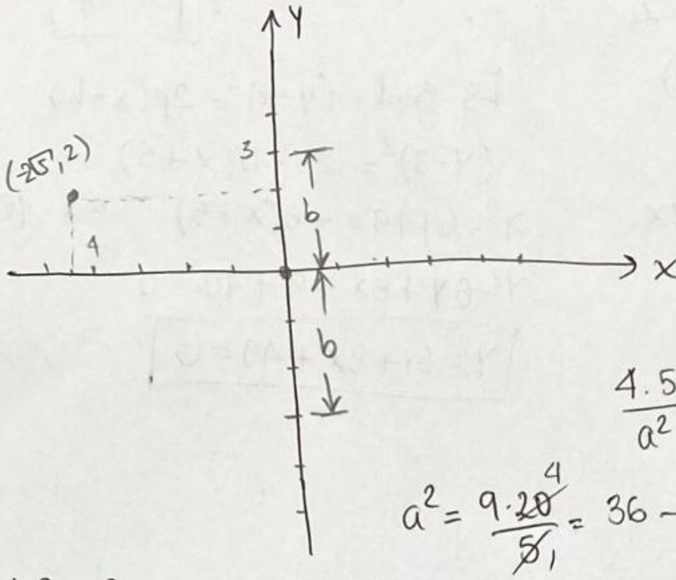
(b) $\left.\begin{array}{l} F_1(-1,0) \rightarrow F_1(-c,0) \\ F_2(1,0) \rightarrow F_2(c,0) \end{array}\right\}$ Eixo maior está sobre o eixo dos x. $\rightarrow$ Centro $(0,0)$.

Vértice $B_1(0,\sqrt{2}) \quad \therefore B_2(0,-\sqrt{2}) \quad , \overline{B_1B_2} = 2\sqrt{2} = 2b \rightarrow b=\sqrt{2}$

$\left.\begin{array}{l} a^2 = b^2+c^2 = (\sqrt{2})^2+1^2 = 2+1 \\ a^2 = 3 \rightarrow a=\sqrt{3} \end{array}\right\} \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \rightarrow \boxed{\dfrac{x^2}{3} + \dfrac{y^2}{2} = 1}$

(c) $C(0,0)$, eixo menor $= 6 = 2b \therefore b = 3 \rightarrow b^2 = 9$

Focos no eixo dos x, e passando pelo ponto $(-2\sqrt{5}, 2)$.



Forma reduzida $\Rightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Substituindo $P(-2\sqrt{5}, 2)$ na Eq. reduzida temos:

$$\frac{(-2\sqrt{5})^2}{a^2} + \frac{2^2}{9} = 1.$$

$$\frac{4.5}{a^2} + \frac{4}{9} = 1 \rightarrow \frac{20}{a^2} = 1 - \frac{4}{9} = \frac{5}{9}$$

$$a^2 = \frac{9 \cdot \cancel{20}^{\,4}}{\cancel{5}_{\,1}} = 36 \rightarrow \underline{a = 6}$$

$$a^2 = b^2 + c^2 \rightarrow c^2 = a^2 - b^2 = 36 - 9 = 27 \rightarrow c = \sqrt{27}$$

$$\therefore \boxed{\frac{x^2}{36} + \frac{y^2}{9} = 1} \text{ ou } \boxed{x^2 - 4y^2 - 36 = 0}$$

4. a) $V(0,0)$, $\underbrace{\text{simetria = eixo da parábola é o eixo dos } y}$. Passa pelo ponto $P(2,-3)$.

Eq. geral $\Rightarrow x^2 = 2p \cdot y$, substituindo o ponto P, temos

$$2^2 = 2 \cdot p \cdot (-3)$$

$$4 = -6p \therefore \boxed{p = -\frac{2}{3}} \quad \therefore x^2 = 2 \cdot \left(\frac{-2}{3}\right) y = \frac{-4}{3} y$$

$$\text{ou } \boxed{x^2 = \frac{-4}{3} y} \text{ ou } \boxed{3x^2 + 4y = 0}$$

b) $V(4,1)$, diretriz $d: y + 3 = 0$.

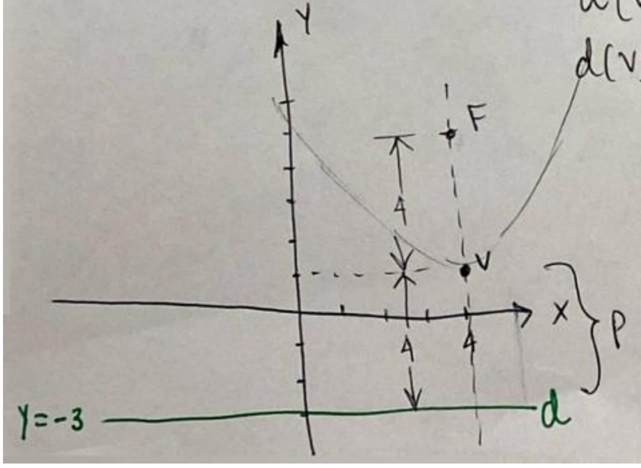
$V(h,k) \neq 0$.

$$\Rightarrow \frac{p}{2} = 4 \rightarrow \underbrace{p = 8 \; (p > 0)}$$

concavidade voltada para cima.

$d(V, d) = 4$

$d(V,F) = 4$



$$\therefore (x-h)^2 = 2p(y-k)$$

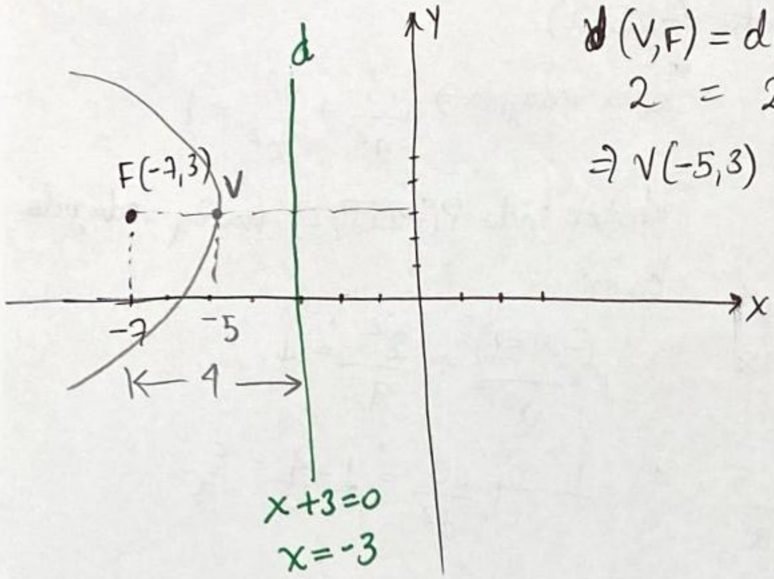
$$(x-4)^2 = 2 \cdot 8(y-1)$$

$$x^2 - 8x + 16 = 16y - 16$$

$$x^2 - 16y - 8x + 16 + 16 = 0$$

$$\boxed{x^2 - 8x - 16y + 32 = 0}$$

c) $F(-7,3)$, diretriz $d: x+3=0 \to x=-3$



$d(V,F) = d(V,d)$

$2 = 2$

$\Rightarrow V(-5,3)$

$p<0 \Rightarrow \frac{p}{2} = -2$

$\therefore \boxed{p=-4}$

Eq. geral: $(y-k)^2 = 2p(x-h)$

$(y-3)^2 = 2\cdot(-4)(x+5)$

$y^2-6y+9 = -8(x+5) = -8x-40$

$y^2-6y+8x+9+40=0$

$\boxed{y^2-6y+8x+49=0}$

5. (a) $(x+4)^2 = -2(y-1)$

Se $t = x+4 \Rightarrow t^2 = -2(y-1)$

$t^2 = -2y+2 \to t^2-2 = -2y \to \boxed{y = 1-\frac{t^2}{2}}$ e $\boxed{x = t-4}$

(b) $9x^2+16y^2=1$

Manipular de modo que os coeficientes de x^2 e y^2 sejam igual a 1.

$9x^2 = \frac{x^2}{\frac{1}{9}}$ e $16y^2 = \frac{y^2}{\frac{1}{16}} \Rightarrow \frac{x^2}{\frac{1}{9}} + \frac{y^2}{\frac{1}{16}} = 1$ Maior denominador: $\frac{1}{9} = a^2$

$\therefore a = \frac{1}{3}$

$b^2 = \frac{1}{16} \to b = \frac{1}{4}$. O centro desta elipse é $C(h,k) \Rightarrow$ as Eqs paramétricas são:

$$\left.\begin{aligned} x &= h + a\cos\theta \\ y &= k + b\,\mathrm{sen}\,\theta \end{aligned}\right\}$$ Eixo maior ∥ a Ox. (Maior denominador no x)

$\therefore x = 0 + \frac{1}{3}\cos\theta \to \boxed{x = \frac{1}{3}\cos\theta}$

$y = 0 + \frac{1}{4}\mathrm{sen}\,\theta \to \boxed{y = \frac{1}{4}\mathrm{sen}\,\theta}$